The purpose of this homework is to determine the efficacy of several advanced filters for denoising. Specifically, we will examine some adaptive filters and some wavelet filters and compare them to conventional methods (Gaussian blurring, median filter). This project is not a typical homework, because the outcome is not known at this time.

Two test images are available: A synthetic image and an image actually taken from a CT. The object is a water-filled glass container. Provided are the unmodified sinogram and the reconstruction with filtered backprojection and the Shepp-Logan kernel (Figure 1 A). A similar object was created synthetically and converted into a sinogram (Figure 1 B). Absorption values, dimensions, and resolution of the synthetic object are very similar to the water-filled glass container. Since the sinogram from the synthetic object is noise-free, non-correlated, Gaussian, zero-mean noise with a standard deviation of $\sigma = 15$ was added to the sinogram in an attempt to simulate as closely as possible the additive detector noise of a CT system.

Image values are either raw ADC output values (proportional to the detector x-ray intensity) or raw absorption values calibrated in cm$^{-1}$. Conversion to Hounsfield units is possible by linear scaling of the image values (Equation 1), and the water area should have a mean value of zero HU. Glass differs between 70kVp and 140kVp, and the synthetic image should correspond to the real 140kVp image.

\[
\begin{align*}
I_{[HU]}(x, y) &= 4762 \cdot (I(x, y) - 0.214) \quad \text{for 140kVp} \\
I_{[HU]}(x, y) &= 3333 \cdot (I(x, y) - 0.300) \quad \text{for 70kVp}
\end{align*}
\] (1)

To determine the quality of the denoised image, we will use a qualitative criterion and three quantitative metrics. All filters should be ranked subjectively based on the overall appearance, more specifically, the balance between noise reduction and edge preservation. In addition, the following metrics are used to determine the quality of the reconstruction:
Figure 1: Sinogram (top) and conventional FBP reconstruction (bottom) of an actual CT image (A) of a glass container filled with water, and (B) a synthetic image that strongly resembles the cross-section of the imaged object, but contains a defined additive, zero-mean noise component. The simulation includes several typical artifacts seen in the real CT image, such as the intensity drop-off in the glass region, and the beam hardening artifacts near the corners.

- A line-spread function serves as a metric of edge degradation. In a horizontal profile approximately in the middle of the reconstructed image, the line spread (in pixels) needs to be determined: The histogram reveals the mode values for glass and water (for example, in a 70kVp reconstruction, the mode value for water is $0.3\text{cm}^{-1}$, and the mode value for glass is $1.13\text{cm}^{-1}$). The line spread can be defined as the 20-80 jump, meaning, the number of pixels needed to rise from 0.466 (20% of the water-glass intensity difference) to 0.964 (80% of the water-glass intensity difference). Interpolation may be necessary. In the 70kVp image, for example, this step spans roughly 1.7 pixels.

- The standard deviation of the water region serves as a metric of noise content. Since water is homogeneous, this area should have a zero standard deviation, but in the 70kVp image, for example, $\sigma \approx 0.021$
is observed.

- The difference image $D(x, y)$, serves as a visual metric of what has been removed by the denoising process. The difference image is calculated as

$$D(x, y) = |I(x, y) - I_{denoised}(x, y)| \quad (2)$$

Particularly revealing is the histogram of $D(x, y)$ as it should show a Gaussian distribution with $\sigma = 15$ when the filter acts perfectly.

If desired, the original (noise-free) synthetic image and its noise-free sinogram are available and can serve as reference. With this reference, the root mean-squared error (RMSE) can be used to obtain an objective metric of the error introduced by the denoising process.

Use at least the following filters:

- Conventional Gaussian blur applied to the reconstructed image
- Conventional 1D Gaussian blur applied to the sinogram before reconstruction. Both Gaussian filters should lead to the same noise component in the water area.
- Adaptive MMSE filter applied to the reconstructed image
- Adaptive MMSE filter (1D) applied to the sinogram before reconstruction
- Anisotropic diffusion applied to the reconstructed image (optimal parameters?)
- Adaptive bilateral filter applied to the reconstructed image (optimal parameters?)

In a second step, we will continue the same analysis with wavelet-based filters. Once again, we will compare 1D coefficient shrinking applied to the sinogram before reconstruction to the same type of wavelet coefficient shrinking in 2D. Some experimentation is necessary, but Donoho’s universal threshold, BayesShrink and SUREShrink should be covered.

**Note: You need to distrust the cimage implementation.** Not all coefficient shrinking functions have been fully tested.

- Choose an intermediate-smoothness wavelet, such as the Daubechies-12 wavelet and a decomposition level of 3. Compare the thresholding methods with transform parameters unchanged.
• Examine the influence of the manual threshold multiplier.

• For two selected threshold methods (the best and the worst), examine how different decomposition levels influences denoising efficacy. It would be interesting to see a diagram that shows the RMSE over the decomposition level.

• In the same manner, examine the choice of wavelet. It is sufficient to examine the Haar wavelet and the Daubechies-series wavelets

Provide your analysis in a concise typewritten report. If you want to include images, keep in mind that the results of the wavelet transform may have floating-point elements. To properly print them, you need to either convert them to 8-bit uchar images, or you may use the colormap function to apply contrast enhancement for printing, then use the "create" function to create a printable 8-bit image.