Assignment 1
P-Control of a Water Bath

In this assignment, you will get familiar with the basic behavior of feedback systems. The goal is to keep a beaker with fluid (water) at a constant temperature. We will use the simplest possible control system, that is, a P-controller. The control action of a P-controller is proportional to the error signal. You will be given a beaker that can be heated by means of four resistors of 10 Ω each. At first, we assume that all heat dissipated by the resistors is delivered into the fluid. You need to build the circuit in the attached schematic. Do not build all of the circuit at once, but rather follow the steps below.

1 Analysis of the Process

The *process* consists of the beaker, the heating resistors, and the voltage-to-current converter built around U3 and Q1. The voltage-to-current converter drives a current I_H through the heating resistors that is proportional to the process input voltage V_P through $I_H = V_P/R_E$ with $R_E = 2.2\Omega$. Total heating power in the heating resistors can be computed from V_P by taking the voltage-to-current converter into consideration:

$$P = 10\Omega \cdot I_H^2 = 2.1\Omega^{-1} \cdot V_P^2 \qquad (1)$$

Figure 1 is provided to allow easier understanding of the electronic circuit that constitutes the sensor, error amplifier, and control action generator. The sensor element is a temperature-dependent zener diode (LM335Z) that is calibrated to precisely 10mV/K. To increase the gain, an amplifier stage (U1) follows. However, at room temperature the sensor delivers 2.9V, and simply multiplying it by 10 would exceed the allowable op-amp voltage range. For this reason, a zero offset is subtracted that is conveniently calibrated to the sensor's output at 0°C (i.e., 2.73V), and the output of U1 delivers a voltage V_T of 100mV per °C.

In the first part of this assignment, we will not consider the sensor and error amplifier. Instead, we will provide a variable voltage V_P as process input (open-loop). Furthermore, we will for now ignore the nonlinear characteristic of the heater element and proceed from V_H to P by using Equation 1.

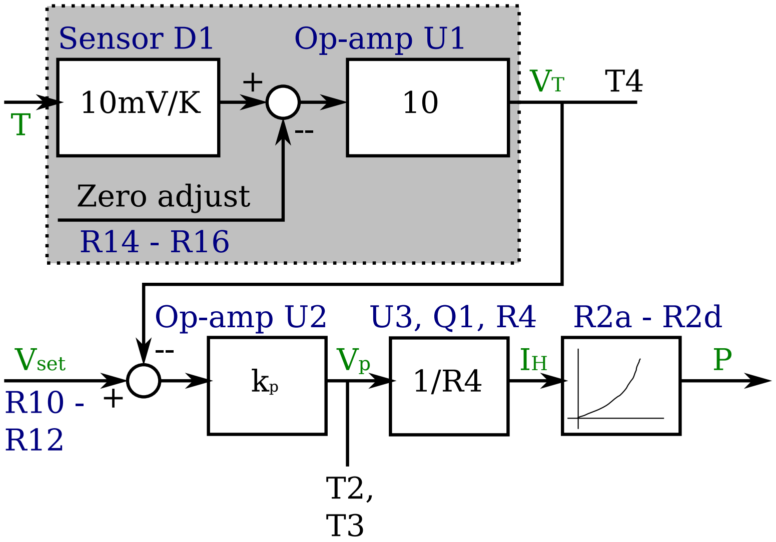

Figure 1: Block diagram of the electric circuit for the sensor and controller. The sensor chip (LM335Z) is calibrated in Kelvin. For this reason a difference amplifier stage subtracts the zero offset and provides a temperature-dependent voltage V_T, calibrated at 0.1V/°C (gray shaded block). This voltage is then subtracted form the setpoint voltage and amplified by a selectable gain k_p in op-amp U2. The control action generator provides a heating power P in dependency of the process input voltage V_P . Note that we consider the last two blocks (1/R4 and the nonlinearity) to be part of the process.

1.1 Step 1, Task 1: Theory

Draw a block diagram of the *process* that consists of two blocks: The beaker (input P, output T) and the heater unit (input V_P, output P). Provide the model for both blocks as follows:

- Heater block: Compute and plot the function P over V_P
- Beaker block: Provide a mathematical model of the temperature T as a function of time, the input power P, and the environmental temperature T_{env} where $P_{env} = k_e \cdot (T - T_{env})$. Here, T is the beaker temperature, P_{env} is the power lost to the environment, and k_e is a proportionality constant that depends on the heat conductivity between beaker and environment. Solve the differential equation and provide one possible solution under the assumption of the application of a step function for P,

$$P(t) = \begin{cases} 0 & \text{for t} < 0 \\ P_{in} & \text{for t} \geq 0 \end{cases} \tag{2}$$

Note that these steps are covered in detail in Chapter 6 of the textbook, and you are encouraged to extract a summary from the chapter.

1.2 Step 1, Task 2: Quantitative Description of the Process

The purpose of this task is to obtain numerical values for all constant parameters of the process. Although some constants can be calculated or estimated, we will in this case apply a step function and measure the temperature response, from which we obtain the constants.

- Determine all constants in the transfer function that you provided in the first task. Start with 100 ml water at room temperature, T_{env}, to fill the beaker (always fill the beaker with 100 ml water). At an arbitrary time $t = 0$ apply a voltage of 10V [1] directly to the heater resistors. You now apply 10W to the process (for voltgaes other than 10V, calculate the corresponding heating power). Monitor, record, and plot the temperature as a function of time, $T(t)$. You need to use a thermometer. Alternatively, use the sensor and convert its output voltage to temperature. You may use data loggers if you have access to them. Figure 2 shows a typical example of a temperature curve.

- Perform a nonlinear function fit and determine the apparent k_w (a constant that depends on the specific heat capacity of the system) and k_e.

- If you wait sufficiently long, you will reach an asymptotic temperature T_∞ for which $P = P_{env}$. Record this temperature as well. From your data (and with the known specific heat capacity of water), compute the heater efficiency η, i.e., how many Watts of power that are dissipated in the resistors are actually available for heating the waterbath.

- After you know the heater efficiency η, compute the corrected k_e . Also determine the time constant τ by computation and verify in your diagram of $T(t)$ that the temperature reached 67% of the difference between initial and final value after time τ.

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[1] You may use higher voltages, provided that you find a suitable power supply. Higher heating power will improve the closed-loop system, although the process (open-loop) time constants do not depend on the heating power

1.3 Step 1, Task 3: Finalizing the Process Electronics

Now add the voltage-current converter with U3 and Q1. Verify the functionality: Does the current through the resistors increase with V_P? Next, apply a variable voltage (potentiometer) to V_P and try to approximately determine the process input voltage $V_{P,37}$ at which you achieve a steady-state water temperature of 37°C.

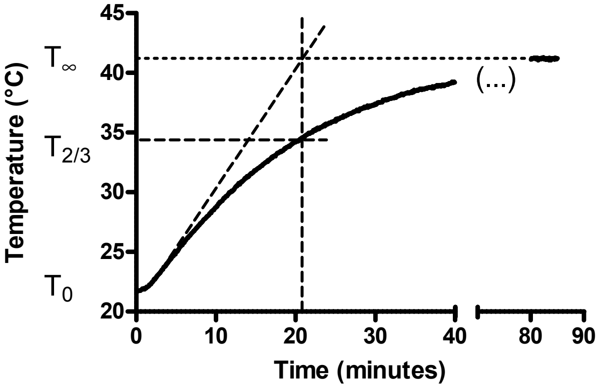

Figure 2: Example for a measured temperature curve. After slightly more than 1 hour, the temperature is fully equilibrated, and T_∞ can be read directly (dotted line). The time constant τ can be found either from the time where the temperature has risen to 2/3 of its final value ($T_{2/3}$) or from the slope neat $t = 0$ (dashed lines).

Alternative approach: Pulse-width modulation (PWM) is a more suitable method to control the heater power from an error voltage. With PWM, the driver transistor is used as a switching element, and losses in Q1 are minimized. Thus, Q1 does not get burning hot as it does with continuous control. Because of the lower losses, PWM is a widespread form of linear actuation in modern control systems. If this interests you, ask for an alternative circuit diagram. It is only marginally more complex than the one provided here.

2 Design of the Controller

2.1 Step 2: Completing the Control Circuit

Connect the sensor amplifier module if you haven't already done so. Make sure the power supply connections are made correctly – otherwise, you may actually damage the module, in which case you need to build the sensor amplifier from scratch. The module has a 10-pin connector (2 rows with 5 pins). The two rows provide mechanical stability, and each pair of pins is electrically connected.

Attach the sensor (note polarity!) and hook up a digital voltmeter to the V_T output. Compare the output to a conventional thermometer. V_T should be calibrated to 0.1V/°C. You can perform some fine-adjustment with the trim potentiometer on the module.

Next, complete the control circuit by adding the error amplifier with U2. Do not yet close the loop at jumper J1. Choose a low amplification: R6 through R9 are all 10kΩ, thus $k_P = 1$. Attach a voltmeter or oscilloscope to both V_T and the setpoint V_S. You should now be able to influence the error signal ϵ with the setpoint potentiometer and the temperature probe. For example, if you minimize ϵ with the setpoint potentiometer and then apply a positive voltage to V_P (i.e., start heating), ϵ should slowly become negative.

Alternative approach: If you use PWM, you use the same sensor module, but the setpoint is included in your PWM oscillator. Therefore, you do not need the explicit error amplifier. To study the open-loop circuit, you need to use a potentiometer that feeds into the comparator and replaces the sensor signal. Adjust the potentiometer to 3.0V (30°C) and verify that increasing the setpoint increases the heating power.

3 Analysis of the Closed-Loop System

3.1 Step 3, Task 1: The Closed-Loop System – Theory

Use the block diagram that you prepared for Step 1 and add the controller blocks to it. The controller should consist of three blocks: The sensor, the sensor amplifier, and the error amplifier (see Figure 1 for additional help).

- Identify the feedforward path and the feedback path. Mark the reference (or setpoint), feedback signal, disturbance, and the controlled output.

- Compute the closed-loop differential equation, and provide numerical values or approximations for all constants.
- Provide a systematic check that all your units match.
- Based on the estimate for V_P to achieve an equilibrium temperature of 37°C, linearize the nonlinear element (see Figure 1).
- Determine the loop gain (but keep k_p rather than using $k_p = 1$).

Alternative approach: If you use PWM, you do not have the nonlinear element. You still need to provide a mathematical description of the closed-loop system, but without the linearization step. In this case, you need to relate the average heating power to the pulse width, and in turn relate the pulse width to the error and setpoint signals. In addition, you need to relate the PWM gain k_p to the amplitude of the triangle generator.

3.2 Step 3, Task 2: Operating the Closed-Loop System

Remove the potentiometer from V_P, set the setpoint voltage to its minimum, and close the loop at J1. Measure the step response of the closed-loop system as follows. Start with water at room temperature. Hook up voltmeters or oscilloscopes to V_T, V_P, and at T5 (the emitter of the power transistor). At an arbitrary time $t = 0$, adjust the potentiometer R12 so that $V_S = 3.7$V (corresponding to a setpoint temperature of 37°C). Monitor and record the three voltages at suitable intervals (e.g., every 60 seconds). Plot the data. However, do not plot the voltage at T5 directly, rather compute the heater power P through Equation 1 and plot V_T, V_P, and P as functions of time. Discuss the results. Does $V_T(t)$ agree with the prediction from Task 1? Do you observe anything special with respect to P? What is your time constant τ, i.e., the time your system needs to reach 63% of its final value? Hint: remember that you are not starting at 0°C. Rather, if your initial temperature is 22 °C, the 63% point is 31.5°C, and τ is the time at which V_T reaches 3.15V. Finally, compute the tangent of V_T at $t = 0$.

3.3 Step 3, Task 3: Control Behavior with Higher k_P

From the results you obtained in Task 2, predict the step response of V_T if k_P was much larger than 1. For this purpose, replace R6 and R8 by 470k. You now have $k_P = 47$ and $V_P = 47\epsilon$. Repeat the experiments from Task 2, starting again with room-temperature water. Plot V_T, V_P, and P as functions of time, and compute τ and the tangent of V_T at $t = 0$. Do the results agree with your prediction? For your answer, consider the difference between the ideal heating power in your linear system and the limits imposed on the heating power by the actual circuit (qualitative answer is sufficient). **Alternative approach:** If you use PWM, you have the ability to continuously adjust k_p through the amplitude of the triangle signal. Choose your higher k_p as a balance between good error amplification and saturation range. What happens to your circuit if you make the triangle amplitude very, very small?

3.4 Step 3, Task 4: Disturbance Rejection

Study the effects of a disturbance. After you reached equilibrium, use a computer fan to direct an air stream onto the beaker, thus increasing k_e. Monitor ϵ and measure or derive heating power P and the new equilibrium temperature T. Compute the new k_e. What is your observation with respect to the new equilibrium temperature?

4 Turn-in and Grading

You are required to turn in two typewritten reports at the assigned due dates. The reports should contain all plots, equations, derivations, and descriptions of the procedures. Each team should turn in one common report, but the report should allow to clearly identify the contributions from the individual team members (e.g. different color, different font, or labeled sections). Try to achieve a balance so that each tem member contributes about equally to the project. The first report covers Steps 1 and 2, and the second report covers Step 3.

Although printed reports are preferred, you may e-mail an electronic version. However, you must use compatible or open formats (such as pdf, dvi, postscript, odt). Word files (.doc, .docx) will be rejected because of their inherent lack of compatibility. Check out http://www.gnu.org/philosophy/no-word-attachments.html for the reasons.

Maximum score points (total of 60) will be assigned as follows:

- STEP 1: Task 1 (5), Task 2 (10), Task 3 (5)
- STEP 2: Task 1 (10)
- STEP 3: Task 1 (5), Task 2 (10), Task 3 (5), Task 4 (5)
- TURNIN: Legible, understandable, well-organized, timely report (5)

The graduate component in Section 4 is pass/fail only and does not add to the score.

5 Additional Hints

Most spreadsheet programs cannot handle curve fitting of a function similar to Equation 3:

$$f(t) = f_0 + S \cdot \left(1 - e^{-\frac{t}{\tau}}\right) \tag{3}$$

Obviously, when you expect a function similar to Equation 3, you cannot simply fit a polynomial or logarithmic function just because the spreadsheet offers that function fit. You will have to choose a suitable software (for example, Labplot, Gnuplot, Grace, or any other statistical data analysis package), or you will have to perform a suitable transformation.

Labplot, Grace, and Gnuplot are Free (as in speech) Software and can be downloaded from the following locations:
Gnuplot: http://www.gnuplot.info/
Grace: http://plasma-gate.weizmann.ac.il/Grace/
Labplot: http://labplot.sourceforge.net/

5.1 Instructions for Linear Curve Fitting

With a suitable transformation, you can determine the constants f_0, S, and τ through linear regression. This is within the capability of conventional spreadsheet programs. To explain the process, let us begin with the first derivative of Equation 3:

$$\dot{f}(t) = \frac{S}{\tau} \cdot \left(1 - e^{-\frac{t}{\tau}}\right) \tag{4}$$

In your data set where you collected data pairs of f_k and t_k, you can approximate the derivative by central differences, that is, $\dot{f}_K \approx (f_{k+1} - f_{k-1})/2\Delta t$ where Δt is the sampling interval. You now have four columns: time t_k, value (temperature or voltage) f_k, time difference $t_{k+1} - t_{k-1}$, and value difference $f_{k+1} - f_{k-1}$. The next transformation is to log-transform the data (Equation 5):

$$\ln \dot{f}(t) = \ln\left(\frac{S}{\tau}\right) - \frac{t}{\tau} \tag{5}$$

In the spreadsheet, add one more column that contains the natural logarithm of the finite difference of the values. Perform a linear fit into the new column over the time column. The slope is $1/\tau$, and S is τ times the intercept raised to the power of e. Finally, f_0 is your initial value of $f(t)$ at $t = 0$. Check your work thoroughly to assure that you obtain reasonably plausible data.

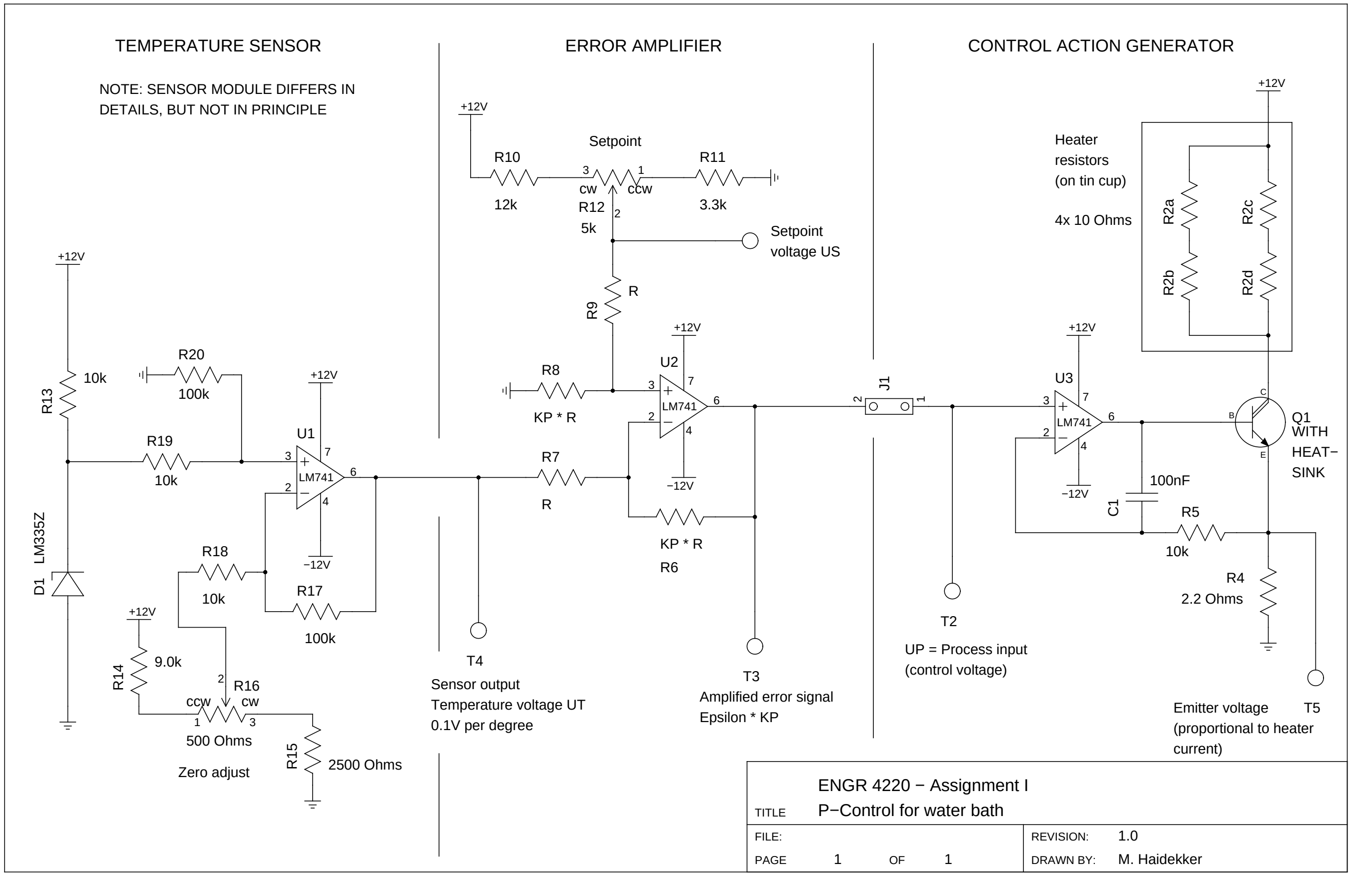

Figure 3: Full circuit schematic of the sensor, controller, and actuator elements.

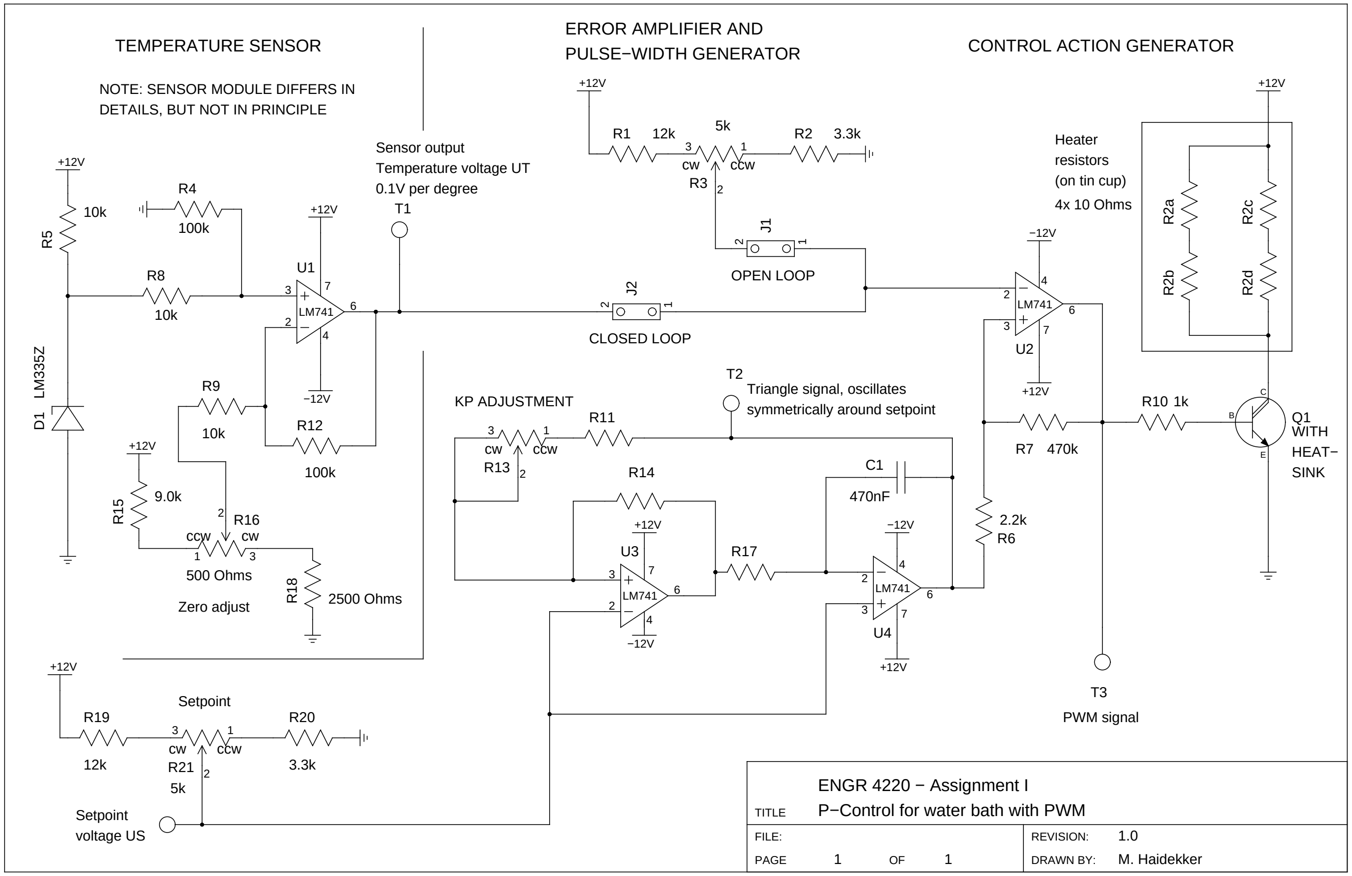

Figure 4: Alternative circuit schematic for PWM control.