ASSIGNMENT 3

In this assignment, you will simulate a linear system and explore controller options. Your primary tool for this assignment will be Scicos. We will try to design a servo system. The servo motor is sketched in Figure 1. A DC motor drives a worm gear which in turn moves an actuator. The worm gear pitch advances the actuator by the distance *a* for each revolution of the motor. If we designate the motor's rotational angle ϕ, then the linear position of the actuator is $x = a\cdot\phi/2\pi$ with the worm gear pitch a=1mm/rad. The motor itself has a rotational inertia of the rotor and worm gear of J. Also, the entire rotating system suffers from friction R_F. A DC motor provides a torque $\tau_M = k_M \cdot I_M$, i.e., the gross torque is proportional to the driving current. Considering inertia and friction, we obtain a balance of torques (Equation 1):

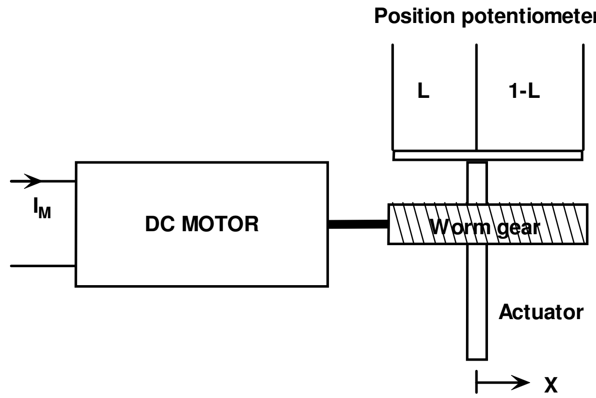

Figure 1: Sketch of the servo motor

$$J \cdot \frac{d\omega}{dt} + R_F \cdot \omega = \tau_M = k_M \cdot I_M \quad \textbf{(1)}$$

Furthermore, the angular position is described by Equation 2 with the initial position ϕ_0.

$$\phi = \int_0^{t'} \omega \, dt + \phi_0 \quad \textbf{(2)}$$

The actuator is attached to a linear potentiometer. To simplify the task, we assume the potentiometer to be normalized (the trace has unity resistance, or 1 Ω). The wiper divides the trace into one part of length L and one of length (1-L). We can apply voltages to the potentiometer in an elegant way: - 1V is applied to the left connector, and +1V is applied to the right connector. The voltage is therefore proportional to x, and the midpoint (x=0) provides a wiper voltage of 0V. At the left and right extremes, the wiper voltage is - 1V and +1V, respectively.

The motor is described by the following constants:

$$J = 0.1\frac{N\,m\,s^2}{rad} \;,\quad R_F = 0.5\frac{N\,m\,s}{rad} \;,\quad k_M = 2.5\frac{Nm}{A} \quad \textbf{(3)}$$

STEP 1: ANALYSIS OF THE PROCESS

Task 1: Draw a Scicos block diagram that describes the servo motor. The process input is the motor current I_M, and the output is the position variable x. Simulate the servo for a step input (100 mA, applied at t=0 with $x_0 = x(t{=}0) = 0$) and plot the position x as a function of time until x reaches 1mm (10^{-3} m).

Task 2: Determine the transfer function of the servo motor in the Laplace domain. Keep in mind that you need to consider the initial position x_0 (related to ϕ_0). Determine the poles and zeroes of the transfer function. Finally, perform an inverse Laplace transform for the same step function for I_M as described above and plot the time function. Compare the result to Task 1.

STEP 2: DESIGN OF A CONTROLLER

The setpoint input of the system is a voltage U_S with $-1\text{V} \le U_S \le +1\text{V}$. The error signal ϵ is obtained by subtracting the servo potentiometer voltage from U_S. Conveniently, the servo potentiometer voltage is described by the position x in volts. Thus, $\epsilon = U_S - x$. A controller with the general transfer function $H(s)$ provides the motor current I_M as a response to the error signal ϵ. The design goal is to reach the desired setpoint position as fast as possible without overshoot.

Task 1: Analyze a P-controller which amplifies the error signal ϵ by a factor of K_P (units: A/m). Derive the Laplace-domain equation for the closed-loop step response of the system when the control voltage steps from 0V to U_S at t=0 and the position at t=0 is x_0. Determine the poles as a function of K_P and the other parameters. What order is the system? Show the location of the closed-loop system poles in the s-plane and compare them to the poles of the open-loop system. Use the final-value theorem to determine the final position. What did you achieve with the P-controller? How does the response change for very large K_P? Finally, determine K_P so that you obtain the fastest step response without overshoot. Describe the pole location for your solution and explain why this location represents the fastest step response without overshoot.

Task 2: Add the P-controller to your block diagram of the servo motor in Scicos. Starting with a very large value for K_P, experiment with different values for K_P in order to obtain a step response without overshoot. It may be advisable to add a second oscilloscope to monitor ω, because ω turns negative after an overshoot. Compare the K_P value to the value you found in Task 1. Now, let the simulation more accurately reflect a real-world controller. A motor driver would have a maximum (saturation) current. In this assignment, the saturation current is ±1.0A, and Scicos can simulate this with the saturation block of the nonlinear palette (Figure 2). The saturation block would be placed right after the K_P amplifier. Simulate the system with saturation. How do the responses to a step stimulus U_S=1V for t >0 differ from the idealized system without saturation?

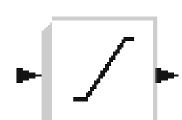

Figure 2: Saturation block

Task 3: Discuss your controller's performance: Can you reach the setpoint? Is the system stable? Are there possible parameter combinations that would make the system unstable? And finally, if you have a disturbance (such as a friction load at the actuator -- which translates into a negative torque in the right-hand side of Equation 1) -- does the controller fully compensate for the disturbance at the K_P value you chose in Task 1?

TURN-IN AND GRADING:

This is not a teamwork assignment. Although you may solve the problems in teamwork, each of you is required to turn in an independent typewritten report at the assigned due date. The report should contain all plots, equations, derivations, and descriptions of the procedures. The report should also contain your Scicos block diagrams.

Although printed reports are preferred, you may e-mail an electronic version. However, you must use compatible or open formats (such as pdf, dvi, postscript, odt). Word files (.doc) will be rejected. Check out http://www.gnu.org/philosophy/no-word-attachments.html for the reasons.

Maximum score points will be assigned as follows:

STEP 1: Task 1 (10), Task 2(5)

STEP 2: Task 1(10), Task 2(5), Task 3 (5)

TURNIN: Legible, understandable, well-organized, timely report (5)

for a total maximum score of 40 points.