Assignment 1
Analysis of System Dynamics

In this assignment, you will examine the behavior of a dynamic system, measure its response and compare your measurements to theoretical predictions. The system is simple pendulum, which is very similar to the spring-mass-damper system that we analyzed in class. To set up your system, you will need a very lightweight, inelastic string (the length of the string is left up to you, however the longer it is the easier it is to perform the experiment), a mass of your choice, a pivot, a tape measure, and a stopwatch. ¹ You will set up your system as shown in Figure 1.

![Figure 1: Sketch of the pendulum. A mass $m$ is attached to a string of length $L$. At the start of the experiment ($t = 0$), the pendulum is moved by the angle $\Theta$ out of the vertical orientation and spontaneously released. The pendulum will now perform damped oscillations and eventually come to rest in the vertical position with $\Theta = 0$.](image)

¹Please make sure that the mass of the bob is much greater than that of the string.
1 Theory

When a pendulum is subject to an angular displacement $\Theta$ from its stable equilibrium position ($\Theta = 0^\circ$), a combination of the restoring force due to gravity and inertia causes it to oscillate about the stable equilibrium point. If we assume a frictionless pivot, negligible drag force, and a light inelastic string ($m_{String} \approx 0$), the system responds to small angular displacement with the simple harmonic motion given in Equation 1:

$$\ddot{\Theta}(t) = -\frac{g}{L} \cdot \Theta(t)$$ (1)

However, if the assumptions stated above do not hold true, the treatment and the actual response of the system become more complex and, for larger deflection angles, nonlinear. For simplicity, it will be important that you use a very light inelastic string and displace the mass by an angle for which the small angle approximation is valid. The small angle approximation is generally accurate as shown in figure 2.

![Figure 2: Plot of sin(\Theta) and \Theta (in radians) over the angle \Theta. It can be seen that the approximation sin \Theta \approx \Theta is very accurate for |\Theta| < \pi/9.](image)

While it is relatively easy to find a string of constant length whose mass is negligible against the bob, and also stay in boundaries of the small angle approximation, it is unrealistic to assume a frictionless environment. If you set up your experiment, you will notice that no matter what you do, your pendulum will come to rest after a number of oscillations. To closely model
reality, we will account for frictional forces acting on the system by adding a velocity-dependent damping term (Equation 2),

\[ F_{Damp}(t) = cL\dot{\Theta}(t) \]  

where \( c \) is the friction coefficient and \( L \) the length of the pendulum.

Set up a model of the system and analyze the theoretical dynamic response as follows:

1. Describe the pendulum dynamics with a differential equation. Include the friction term. Hint: Since you release the pendulum at \( t = 0 \), you will obtain a homogeneous differential equation with initial condition. The input variable is your initial condition, that is, \( \Theta_0 = \Theta(t = 0) \), and your output variable is \( \Theta(t) \).

2. Show that the solution for the differential equation is a harmonic oscillation when \( c = 0 \). What are the amplitude and the frequency of the harmonic oscillation?

3. Provide the Laplace-transform of the differential equation in its general form with \( c \neq 0 \), identify its characteristic equation and the transfer function when the initial condition is considered to be the input to the system.

4. Determine the poles and zeros (if any) of the transfer function and plot them in the \( s \)-plane. Do the locations of the poles relate to any element of the dynamic behavior?

5. Normalize the characteristic equation to the standard form \( q(s) = s^2 + 2\zeta\omega s + \omega^2 \) and determine the damping coefficient (\( \zeta \)) from \( c \). Does this make sense? Why/why not?

6. From transfer function and initial condition (assume \( \Theta_0 = \pi/6 \)), determine the inverse Laplace transform of the system response \( \Theta(s) \) and plot the time-domain response as \( \Theta(t) \). Use your values for \( L \) and \( m \), and assume \( c = L/100 \). Note that you may not need all of these coefficients.

7. From your time-domain plot, determine the period of oscillation and the decay constant of the exponential decay.

8. How would the dynamic response change if the following were changed:
• Mass of the bob increases
• Length of the string increases
• Size and shape of the bob?

Explain the reasons for your answers.

2 Experimental

1. Set up your pendulum system. Ideally, find a high-bay area with no fragile stuff nearby. Also, try to mark off different angles on a nearby wall.

2. Determine the length from the center of mass of the bob to the pivot.

3. Set the bob to its initial position ($\Theta_0$) as shown in figure 1.

4. Release the bob. Immediately after the bob is released note the peak of the displacement $\Theta$ and thus the amplitude. By using a stopwatch, determine the period of the oscillation. Moreover, determine the decay of the amplitude every few cycles (here you need those angular marks!).

5. From your data, determine the damping coefficient $c$. You can either fit an exponential function to the peak values vs time, or you can use the "logarithmic decrement" method. Note: Either of these two methods will allow you to experimentally determine the damping coefficient.

6. Recalculate your model in Section 1.6. and use the experimental value for $c$. Plot $\Theta(t)$ with the new value of $c$.

7. How well do period of oscillation and damping match the experimental data? Explain any major deviations!

Turnin and Grading

You are required to turn in one typewritten report at the assigned due date. The report should contain all plots, equations, derivations, and descriptions of the procedures. Each team should turn in one common report, but the report should allow to clearly identify the contributions from the individual team members (e.g. different color, different font, or labeled sections). Try to achieve a balance so that each team member contributes about equally to the project.

Maximum score points (total of 60) will be assigned as follows:

• EXPERIMENTAL: Task 1 (2), Task 2 (2), Task 3 (1), Task 4 (5), Task 5 (5), Task 6 (5), Task 7 (5)

• TURNIN: Legible, understandable, well-organized, timely report (5)

Although printed reports are preferred, you may e-mail an electronic version. However, you must use compatible or open formats (such as pdf, dvi, postscript, odt). Word files (.doc, .docx) will be rejected because of their inherent lack of compatibility. Check out http://www.gnu.org/philosophy/no-word-attachments.html for the reasons.